90642

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2003

90642 Calculate confidence intervals for population parameters.

Credits: Three

Answer ALL questions in the spaces provided in this booklet.

Show ALL working for ALL questions.

Check that this booklet has pages 2–5 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only **Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Calculate confidence intervals for population parameters.	Demonstrate an understanding of confidence intervals.	Analyse estimates of population parameters.
Overall Level of Performance		

You are advised to spend 40 minutes answering the questions in this booklet.

Show ALL working.

QUESTION ONE

A pet food company bought a new machine to fill bags of dog food that would be labelled and sold as 3kg bags.

A random sample of 150 bags was taken. Each bag was weighed to test whether the bags met the specified weight. The sample was found to have a mean weight of 3.128 kg and a standard deviation of 0.098 kg.

Determine a 99% confidence interval for the mean weight of all bags filled by this machine.

QUESTION TWO

A random sample of 70 bags of the company's dry cat food was taken. Each bag was weighed to test whether the bags met specifications regarding their weight.

The sample gave a mean of 1020 g and a standard deviation of 8 g.

Find a 90% confidence interval for the mean weight of all bags of dry cat food.

QUESTION THREE

The company launched a new product. Some time after the launch, a survey was conducted to find the proportion of pet owners who had heard of the new product. Of 500 pet owners surveyed, 147 said they had heard of the new product.

(a) Use the survey results to find a 95% confidence interval for the proportion of all pet owners who would have heard of the new product.

(b) Explain the meaning of the confidence interval found in part (a).

QUESTION FOUR

A 98% confidence interval for the proportion of protein in pellets of pet food was calculated to be $0.25 < p < 0.31$.

What was the size of the sample that generated this confidence interval?

QUESTION FIVE

The company markets its pet food under two brand names, Happy Chappy and Wow! A sample of 400 cans of Happy Chappy pet food was taken and was found to have a mean weight of 512 g and a standard deviation of 10 g.

A sample of 350 cans of Wow! pet food was taken and was found to have a mean weight of 513 g and a standard deviation of 11 g.

(a) Calculate a 95% confidence interval for the difference between the population means for the two brands.

(b) Does your confidence interval support the proposition that there is a difference between the population means for the two brands?

Justify your answer.

(c) Another sample of Happy Chappy and of Wow! pet food was taken. A 95% confidence interval for the difference between the two means was calculated and it was found to be half the width of the interval calculated in part (a).

What is the most likely cause of the new confidence interval being narrower?

Justify your answer, including references to the standard error.

QUESTION SIX

Large scale market research has shown that, for all households buying pet food, the company has a 38.4% market share. In an effort to increase its market share, the company launched an advertising campaign. At the conclusion of the campaign, the effectiveness of the campaign was tested by polling a random sample of 500 households. Of the households surveyed, 40.7% bought the company's products.

Is it likely that there has been an actual increase in the company's market share for all households buying pet food?

Use statistical reasoning to support your answer.

Assessment Schedule (sample)

Statistics and Modelling: Calculate confidence intervals for population parameters (90642)

	Achievement Criteria	Qn No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT	Calculate confidence intervals for population parameters.	1	standard error $= \frac{0.098}{\sqrt{150}} = 0.008$ fi $3.107 < m < 3.149$ kg	A	Or equivalent	***Achievement:*** Two of Code A
		2	standard error $= \frac{8}{\sqrt{70}} = 0.956$ fi $1018.4 < m < 1021.6$ g	A	Or equivalent	
		3(a)	standard error $= \sqrt{\frac{0.294 \times 0.706}{500}} = 0.0204$ fi $0.254 < p < 0.334$	A	Or equivalent	Incorrect rounding will not be penalised.

	Achievement Criteria	Qn No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT WITH MERIT	Demonstrate an understanding of confidence intervals.	3(b)	If we were to repeatedly find confidence intervals using the same sampling process, in the long run about 95% of the confidence intervals found would enclose the true proportion.	M	Or equivalent	***Merit:*** Achievement **plus** Three of Code M **or** Four of Code M
		4	$2.326 \times \sqrt{\frac{0.28 \times 0.72}{n}} < 0.03$ Minimum sample size = 1212	M	Consistent with z value	
		5(a)	standard error $= \sqrt{\frac{10^2}{400} + \frac{11^2}{350}}$ $= 0.772$ $-0.51 < \mu_1 - \mu_2 < 2.51$ g	M	Or equivalent	
		5(b)	Because the interval encloses 0, it is most unlikely that there is a difference between the two means.	M	Or equivalent	

	Achievement Criteria	Qn No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT WITH EXCELLENCE	Analyse estimates of population parameters.	5(c) 6	There has been an increase in the sample size, which has made the standard error smaller, thereby making the confidence interval narrower. $q_P = \sqrt{\frac{p(1-p)}{n}} = 0.02175$ (based on p = 0.384) = 0.02194 (based on p = 0.407) One of: (i) the sample proportion is only 1.05 q_P or 1.06 q_P from the mean of the sampling distribution, so is not unusual, and there is not strong evidence to suggest change of population proportion (market share) (ii) P(p > 0.407) = 0.15 and there is not strong evidence to suggest change of population proportion (market share) (iii) 95% CI for population proportion (market share) after ad campaign is $0.364 < \pi < 0.450$, and 38.4% is in this interval, so there is not strong evidence to suggest change of population proportion (market share) **or** 90% CI for population proportion (market share) after ad campaign is $0.370 < \pi < 0.443$, and 38.4% is in this interval, so there is not strong evidence to suggest change of population proportion (market share).	E E E E	Looking for two parts to explanation (underlined) Answer must include a calculation involving the sampling distribution of the proportion. Must draw the conclusion that the sample prop is not unusual and there is not strong evidence that the ad campaign met its aim (to increase market share). Other ways of expressing the reasoning and the conclusion that meet the specifications above are acceptable.	***Excellence:*** Merit **plus** Three of Code E* * The three of code E is from the two parts in 5(b) and the calculation and the conclusion in 6.